CMP ASSESSMENT COMMITTEE REPORT #2

ASSESSING KNOWLEDGE OF INSTRUCTIONAL STRATEGIES

1. Introduction

Last year, the CMP Assessment Committee issued its initial report responding to AB 1734's first CSMP goal: "Develop and enhance teachers' subject matter (i.e., content) knowledge)." That report sought to provide a conceptual framework for understanding content and pedagogical content knowledge within a mathematical context, offered some general advice regarding assessment, and furnished information for employing rich problems and concept maps as adjuncts to standardized tests.

Committee Report #2 is responsive to AB 1734's second goal: "Develop and enhance teachers' instructional strategies to improve the academic performance of their students." How might CMP Sites assess the progress of their teacher-participants in meeting this goal? While it is clear that real time observation techniques (whether in the classroom or during teacher to teacher presentations) can provide important data, the Committee also believes that judicious use of soundly constructed, paper and pencil assessment items can also provide useful, evaluative information . Therefore, our objective for this report is to present a functional taxonomy for thinking about assessing instructional strategies for teachers of mathematics, and to link that taxonomy with specific examples and resources.

2. The Taxonomy

As with any useful taxonomy, we make no claim that the following is either mutually exclusive or jointly exhaustive. Rather, we present it as a tool, a framework, for beginning to think about the multiple ways that teachers must incorporate diverse instructional strategies into their classrooms.

I. Curriculum-centered

A. Develop the scope and sequence for the classroom presentation of a specific, standards-based, mathematical concept

B. Identify several models of representation for a specific, standards-based, mathematical concept

C. Identify several problem solving strategies helpful in solving specific, standards-based, problem types (e.g., addition of fractions, linear equations)

D. Provide connections between and among different mathematical topics and between and among mathematics and the physical world that it models

II. Student-centered

A. Identify and remediate student misconceptions

B. Identify prerequisite knowledge gaps that may be hindering student performance and design remediations

C. Demonstrate strategies for answering student generated questions

D. Describe and explain knowledge of various student assessment techniques and strategies

The following is hardly intended to be comprehensive. Each taxonomic node is briefly explained and a sample question and short answer are supplied. As with the taxonomy itself, our goal is only to present a catalyst to discussion. We hope that individual Sites will extend our thinking, deepen it, and share their work with the entire CMP community.

3. The Examples - Curriculum-centered

I.A. Develop the scope and sequence for the classroom presentation of a specific, standards-based, mathematical concept

Scope and sequence is always a time based concept and may apply equally well to the entire K-12 mathematics curriculum as to a daily lesson plan. Of course, in the CMP context of teacher professional development, lesson plans are an appropriate time frame and several sites include the development of such plans in their activities. For the purposes of assessment, we recommend a very targeted prompt drawn directly from the standards. For example,

Q: Describe how you would present an introductory lesson to third grade students designed to help them understand the place value of whole numbers. Your answer should include the mathematical goals of the lesson, objectives for reaching those goals, strategies for motivating student interest in the place value concept, and sample exercises to reinforce and assess student understanding.

A. Obviously, there is no single, correct answer. Still, we should expect laudable answers to contain the following information/insights: A statement, understandable to third graders, defining whole numbers and place value. Objectives linked to the five substandards (e.g., compare and order whole numbers to 10,000) for third grade number sense. Motivational stories or real world examples demonstrating the importance of place value. Exercises (e.g., using expanded notation to represent numbers, standard 1.5) to reinforce and assess understanding.

I. B. Identify several models of representation for a specific, standards-based, mathematical concept

Fractions play a substantial role in the number sense strand for grades 2-7. We should expect any teacher of these grades to be able to model fractions in several different ways and to be able to explain the connections/relationships among/between the models.

Q. Fractions are a key mathematical concept from grade two forward. Provide a precise mathematical definition of 'fraction', give five, separate models for representing the fraction concept and explain their connections.

A. Fractions are ratios (or quotients) of two numbers. They may be modeled as parts of a whole, parts of a set, indicated division of whole numbers and quantities (and measures) between whole numbers on a number line, decimals, and percentages.

I.C. Identify several problem solving strategies helpful in answering specific, standards-based, problem types (e.g., addition of fractions, linear equations)

The study of linear functions begins in grade 5 and plays an important role in the Algebra I curriculum. Solving pairs of linear equations is a necessary condition of succeeding in Algebra and teachers must be able to demonstrate several strategies for dealing with these types of problems.

Q. Given the pair of linear equations $x + 2y=7$ and $y=2x+1$, describe three separate strategies for obtaining a solution.

A. Linear equations may be solved graphically, by identifying the unique point of intersection (1,3); by substitution, by expressing one unknown in terms of the other (in this case, by substituting $2x+1$ for y in the first equation and solving for x); or by addition or subtraction, in this case by multiplying the second equation by 2 and adding it to the first in order to solve for x).

I.D. Provide connections between and among different mathematical topics and between and among mathematics and the physical world that it models

Virtually every grade level contains opportunities to link mathematics to the world that it models. For example, the kindergarten standard for measurement and geometry 1.2 deals with clocks and calendars, the third grade standard for measurement and geometry 1.1 requires students to choose appropriate units and tools in order to estimate length, liquid volume, and weight/mass, and the trigonometry standard has students applying trigonometry in a variety of applications. As such, the direct application of this taxonomic point is identical with I.A. There is, however, another, indirect sense for I.D. and that is the connection between mathematics and virtually every other field of knowledge -- art, music, science, language, social studies all have direct and indirect connections to mathematics. The knowledgeable teacher may use these connections in a variety of ways: to motivate students, to provide concrete examples of abstract mathematics, to encourage an interdisciplinary curriculum.

Q. Proportion and symmetry are two central mathematical and artistic concepts. For your specific grade level, demonstrate how you might use artistic activities to help teach one of the mathematical concepts.

A. For example, the sixth grade number sense 1.3 standard requires the use of proportions to solve problems (e.g., find the length of a side of a polygon similar to a known polygon). Instead of approaching this as problem in abstract mathematics, it could be embedded as part of an activity to design a swimming pool. The diagram of the pool shape (some polygon) would be similar to the actual pool (in fact, the diagram would be its model) and, depending on the size relationship between the pool and model drawing, students would be expected to figure out the exact size of the actual pool's sides. Additional activities might include studying other types of architectural scale drawings (blueprints) and a general discussion of the notion of model.

4. The Examples - Student-centered

II.A. Identify and remediate student misconceptions

Effective teaching requires much more than excellent presentation skills ("the sage on the stage"). Effective teachers need to excel in two way modes of communication: Listening/observing is as important as speaking, and diagnosis of student mistakes is a central outcome of this two way communication and essential to student remediation. According to Robert Ashlock: "Diagnostic teaching involves careful observation. It attempts to determine what individuals are actually learning -- the concepts they are truly learning and the procedures they are really employing. To teach diagnostically, you must adapt your instruction to what you observe and what you learn about each student." The Ashlock book sent to all CMP Sites, Error Patterns in Computation, contains many chapters (grouped by mathematical topics) giving examples of student mistakes followed by explanations and remediations.

II.B. Identify prerequisite knowledge gaps that may be hindering student performance and design remediations

In many ways, II.B. is a subset of II.A; yet, it is such a frequent source of student error that it seems deserving of its own taxonomic category. For example, students may have problems computing because they have failed to fully learn their basic arithmetic facts. Students may fail to solve percentage problems because they haven't mastered the concept of place value. Students may err in trying to solve algebraic equations because they haven't mastered the rules for manipulating exponents. Such "simple" explanations of student errors are qualitatively different than student misconceptions based on (e.g., overgeneralization or oversimplification) and may be, in fact, easier to remediate once the proper diagnosis is made.

Q. Choose a specific strand from your grade level and create several examples of erroneous student work that can be explained by an existing prerequisite knowledge gap.

A. For example, the fifth grade geometry and measurement standard 1.0 requires students to compute the areas of simple objects. Suppose a student's paper looked like this:

1. A square of side 2 inches has what perimeter and what area? P = 8 inches, A = 4 inches

2. A rectangle of length 2 inches and width 4 inches has what perimeter and area? P = 12 inches, A = 8 inches

Clearly, the arithmetic operations are correct. This student does not understand the concept of area (square units) that was taught as standard grade 3, measurement and geometry 1.2, grade 4, algebra and functions 1.4, and grade 4, measurement and geometry, 1.0-1.4.

II.C. Demonstrate strategies for answering student generated questions

Mathematics curricula often focusses on the why and what questions, yet students may be most motivated (or puzzled) by the how and what if questions:

1. Why? Why is this concept or topic important?

2. What? What is the concept?

3. How? How is this concept used?

4. What if? What if we modify the concept?

As mentioned above in the context of diagnosis, good teaching requires superior communication skills and responding to (or encouraging) student questions is one of the basic communication competencies. In order to help expose teachers to the vast array of questions that students may ask, we have secured copies of Crouse's and Sloyer's <u>Mathematical Questions from the Classroom</u> for each site.

Q. For your grade level, provide examples of each of the four types of questions; provide answers as well.

A. 1. Why bother with absolute value? After all, you spent a lot of time explaining that numbers should be signed, why now disregard the sign for ‘absolute’ magnitude? See the answer on pg. 180 of Crouse and Sloyer.

2. Since the addition of fractions requires a common denominator, and since you told us that multiplication was like serial addition, why doesn’t the multiplication of fractions require a common denominator? See the answer on pg. 15 of Crouse and Sloyer

3. If mathematics is more than ‘head games’ it must have some use. But these imaginary and complex numbers just seem useful to mathematicians so they can do more mathematics. How do non-mathematicians use complex numbers to solve real world problems? See the answer on pg. 227 of Crouse and Sloyer.

4. What if dividing zero by itself is defined as one? After all, 1/1=1, 2/2=1, ... So why can’t 0/0=1? See the answer on pg, 43 of Crouse and Sloyer.

II.D. Describe and explain knowledge of various student assessment techniques and strategies

There are almost as many forms of assessment as there are standards: formal and informal, formative and summative, analytic and holistic, grading and ranking, et. al. Equally, there are almost as many assessment techniques that a teacher may use to gather data of student performance: open and closed problems, standardized tests, classroom observation, math journals, concept maps, et. al. We know of no research that demonstrates the absolute supremacy of one technique over all others. In fact, there are many reasons to believe that a well-balanced approach to classroom assessment is the best strategy to adopt: it enables students to do mathematics in several contexts, it exercises students’ different communication skills, and it gives all students the best opportunity to demonstrate their mathematical knowledge. Assessment is part of teaching and good teachers need to know and understand many assessment techniques.

Q. Choose a given standard for your grade level and describe and explain three different techniques (assessment activities) that you might use to assess student understanding.

A. For example, the grade seven measurement and geometry standard 3.0 requires that students know the Pythagorean Theorem. Assessment might be done through a series of closed problems requiring students to find the lengths of missing sides of right triangles. Alternatively, students might be given rich, open problems that require the application of the Pythagorean Theorem in order to determine the solution. Finally, students might be expected to demonstrate (prove) the Pythagorean Theorem.

5. Working with Instructional Strategies

The Committee believes that assessing teacher-participant's fluency with instructional strategies may form part of a well-balanced approach to Site assessment. While it can be informative regarding AB 1734's initial goal, to enhance content knowledge, it cannot replace the types of assessment (e.g., standardized tests and rich problems) that Site's conducted in 2000. Alternatively, those tests and problems shed little light on AB 1734's second goal, to develop teachers' instructional strategies. It is our hope that Sites will supplement their 2000 testing program with problems of the types we have discussed here -- both so that teacher-participant's attention is focussed on this second goal and also so that CMP has evidence of accountability to the AB 1734 mandates.

Finally, it is obvious that the types of problems discussed above require rubric scoring. By way of example, we offer the rubric used with the Praxis Mathematics Pedagogy Exam (one caveat: given our penchant for brevity, we don't expect our sample answers to attain a score of 5, they are not offered as detailed exemplars but as samples of the direction answers may take):

Generally, a response that does not demonstrate an understanding of the mathematics to be presented CANNOT receive a score above 2, regardless of any criteria for higher scores met by the response.

Requirements for a score of 5:

Clearly demonstrates an understanding of the mathematics to be presented
Clearly explains how to present the mathematics to students in a way that is likely to achieve the desired goal(s)
Gives a clear and complete response
Develops the mathematics in a way that is well motivated (that is, students can clearly see why the mathematics being presented is worth studying and/or see the mathematics as the logical consequence of previously studied mathematics

Requirements for a score of 4:

Clearly demonstrates an understanding of the mathematics to be presented but may have a notational error or minor mathematical misstatement
Explains how to present the mathematics to students in a way that can reasonably be expected to achieve the desired goal(s)
Either gives an almost complete response and a well motivated development of the

material OR gives a complete response and a fairly well motivated development of the material

Requirements for a score of 3:

Demonstrates an understanding of the mathematics to be presented
Indicates how to present the mathematics to students in a way that can reasonably be expected to achieve the desired goal(s)
Either gives an almost complete response and a well motivated development of the material OR gives a complete response and a fairly well motivated development of the material

Requirements for a score of 2:

Either demonstrates a limited understanding of the mathematics to be presented (and may or may not indicate how to teach the mathematics to students in a way that is likely to achieve the desired goal(s) OR demonstrates an understanding of the mathematics but gives little or no indication of how to present the mathematics to students in a way that is likely to achieve the desired goal(s)
Gives an unclear and incomplete response

Requirements for a score of 1:

Either demonstrates a very limited understanding of the mathematics to be presented OR fails to discuss the mathematics at all